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Is the Zee model neutrino mass matrix ruled out?

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Abstract. A very economic model of generating small neutrino masses is the Zee model. This model has been studied extensively in the literature with most of the studies concentrated on the simplest version of the model, where all diagonal entries in the mass matrix are zero. SNO, and KamLAND data disfavor this simple version, but only when one also combines information from atmospheric and K2K data can one rule out this model with a high confidence level. We show that the simplest version of the Zee model is ruled out at 3σ level. The original Zee model, however, contains more than enough freedom to satisfy constraints from the data. We propose a new form of mass matrix by a naturalness consideration. This new form of the mass matrix is consistent with all experimental data. It predicts that $m_{\nu_3} = 0$, and $\tan^2\theta_{\rm solar}$ increases and $\sin^2\theta_{\rm atm}$ decreases with $|V_{e3}|$. For $\tan^2\theta_{\rm solar}$ and $\sin^2\theta_{\rm atm}$ to be in their 1σ allowed regions, $|V_{e3}|$ is sizeable but can be set below the 90% C.L. upper bound.

There are abundant data [1–6] from solar, atmospheric, laboratory and recent long baseline (K2K and KamLAND) experiments on neutrino mass and mixing. It is certain that some of the neutrinos have non-zero masses, and also different neutrino species mix with each other. In the minimal standard model (SM) in which there is just one Higgs doublet in the scalar sector and there are no right-handed neutrinos, neutrinos are massless. In order to have non-zero neutrino masses and mixing, one must go beyond the minimal SM.

There are various possible ways to generate neutrino masses. A very economic way of generating neutrino masses is to introduce a charged scalar and an additional Higgs doublet into the minimal SM as proposed by Zee [7]. The Zee model provides a natural mechanism to generate small neutrino masses because they can only be induced at the loop level, and it also suggests special forms for the mass matrix. If one imposes a discrete symmetry such that only one of the Higgs doublets couples to the leptons, as suggested by Wolfenstein [8], one obtains a simple mass matrix with all diagonal entries zero. We will refer to this simple version as the Zee-Wolfenstein model. This model has been studied extensively in the literature [7–12]. In this paper we further study the Zee model using the most recent experimental data. We show that the Zee-Wolfenstein model is ruled out at the 99.73% (3σ) C.L. However, the original Zee model contains more than enough freedom to satisfy experimental constraints [12]. We propose a new form of neutrino mass matrix resulting from a naturalness condition. This model predicts that $m_{\nu_3} = 0$, and $\tan^2\theta_{\rm solar}$ increases with $|V_{e3}|$. For the best fit value of 0.4 for $\tan^2\theta_{\rm solar}$, $|V_{e3}|$ is sizeable but below the 3σ upper bound.

The Zee model contains, in addition to the gauge bosons and the minimal fermion contents, a singlet scalar h and two Higgs doublets $\phi_{1,2}$ transforming under the SM gauge group $SU(3)_C \times SU(2)_{\rm L} \times U(1)_Y$ as $(1,1,1)$ and $(1,2,-1/2)$. With these particles it is not possible to have tree level neutrino masses from a renormalizeable Lagrangian, but it is possible at one loop level. The relevant terms in the Lagrangian are [7]

$$L = -\bar{l}_{d\rm R}\tilde{f}^{\phi,db}_{\gamma}\phi^i_{\gamma}\psi^j_{b\rm L}\epsilon_{ij} - \psi^{{\rm T}i}_{a\rm L}\tilde{f}^{ab}C\psi^j_{b\rm L}\epsilon_{ij}h \\ -M^{\alpha\beta}\phi^i_{\alpha}\phi^j_{\beta}\epsilon_{ij}h + \text{H.C.}, \quad (1)$$

where $\psi^i_{a\rm L} = (\nu_{a\rm L}, e_{a\rm L})$ and $l_{a\rm R}$ are the left- and right-handed leptons with "a" the generation index and "i, j" the $SU(2)_{\rm L}$ indices. ϵ_{ij} is the anti-symmetric symbol. C is the Dirac charge conjugation matrix. $\tilde{f}^{\phi,ab}_{\gamma}$ are the Yukawa couplings responsible for the charged lepton masses. $\tilde{f}^{ab}$ is an anti-symmetric matrix in the generation indices a and b due to Fermi statistics.

The mass matrix $\tilde{m}$ for the charged leptons is given by $\tilde{m} = (v_1\tilde{f}^{\phi}_1 + v_2\tilde{f}^{\phi}_2) = v(\sin\beta\tilde{f}^{\phi}_1 + \cos\beta\tilde{f}^{\phi}_2)$. Here $v_\gamma = \langle\phi_\gamma\rangle$ are the vacuum expectation values (VEV), $v = \sqrt{v_1^2 + v_2^2} = 174\,\text{GeV}$ and $\tan\beta = v_1/v_2$. The mass matrix $\tilde{m}$ can be diagonalized to obtain the eigen-mass matrix $m = \text{Diag}(m_e,\ m_\mu,\ m_\tau)$ by a bi-unitary transformation multiplying two unitary matrices $V_{\rm L,R}$ from left and right, $m = V_{\rm R}\tilde{m}V^\dagger_{\rm L}$.

The linear combination $\phi^-_W = \cos\beta\phi^-_1 + \sin\beta\phi^-_2$ is "eaten" by W^-. The physical combination which mixes

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with h is $\phi^- = \sin\beta\phi_1^- - \cos\beta\phi_2^-$. We indicate the two mass eigenstates of the masses M_1 and M_2 for the charged scalars by $h_1 = \cos\theta_Z h - \sin\theta_Z \phi^+$ and $h_2 = \sin\theta_Z h + \cos\theta_Z\phi^+$. Here $\sin\theta_Z$ is proportional to $M_{\alpha\beta}$ characterizing the strength of the h–ϕ^+ mixing.

The terms responsible for the neutrino mass generation in the previous equation, in the mass eigenstates basis for the charged lepton and scalar fields, can be written as

$$\begin{aligned} L &= -\bar{E}_{\mathrm{R}} m E_{\mathrm{L}} \\ &- \bar{E}_{\mathrm{R}}\left(\frac{1}{v\tan\beta}m - \frac{1}{\sin\beta}f_2^{\phi}\right)\nu_{\mathrm{L}}(\sin\theta_Z h_1^{\dagger} - \cos\theta_Z h_2^{\dagger}) \\ &- 2\nu_{\mathrm{L}}^{\mathrm{T}} f C E_{\mathrm{L}}(\cos\theta_Z h_1 + \sin\theta_Z h_2) + \dots , \end{aligned} \tag{2}$$

where $f_\gamma^{\phi} = (f_\gamma^{\phi,ab}) = V_{\mathrm{R}}\tilde{f}_\gamma^{\phi}V_{\mathrm{L}}^{\dagger}$, $f = (f^{ab}) = V_{\mathrm{L}}^{*}\tilde{f}V_{\mathrm{L}}^{\dagger}$, $E_{\mathrm{L,R}} = (e,\ \mu,\ \tau)_{\mathrm{L,R}}$, and $\nu_{\mathrm{L}} = (\nu_1, \nu_2, \nu_3)_{\mathrm{L}}$.

Exchange of charged scalars $h_{1,2}$ and charged leptons at one loop level, a Majorana neutrino mass term $L_m = (1/2)\nu_{\mathrm{L}}^{\mathrm{T}} M_\nu C \nu_{\mathrm{L}}$, can be generated with

$$M_\nu = A\left[(fm^2 + m^2 f^{\mathrm{T}}) - \frac{v}{\cos\beta}(fmf_2^{\phi} + f_2^{\phi\mathrm{T}} m f^{\mathrm{T}})\right], \tag{3}$$

where $A = \sin(2\theta_Z)\log(M_2^2/M_1^2)/(16\pi^2 v\tan\beta)$ which is of order $O(10^{-5})$ if the $\sin(2\theta_Z)$ and $\tan\beta$ terms are both of order 1. This is the general mass matrix in the Zee model [12]. The mixing matrix is the unitary matrix V which diagonalizes the mass matrix and is defined by $D = V^{\mathrm{T}} M_\nu V$, with $D = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$.

The present experimental data on neutrino masses and mixing angles can be summarized as follows [13,14]. The 3σ allowed ranges for the mass-squared differences are constrained to be $1.6\times10^{-3}\,\mathrm{eV}^2 \le |\Delta m^2_{\mathrm{atm}}| \le 4.8\times10^{-3}\,\mathrm{eV}^2$, and $4.7\times10^{-5}\,\mathrm{eV}^2 \le \Delta m^2_{\mathrm{solar}} \le 1.7\times10^{-4}\,\mathrm{eV}^2$, with the best fit values given by $|\Delta m^2_{\mathrm{atm}}| = 2.5\times10^{-3}\,\mathrm{eV}^2$, and $\Delta m^2_{\mathrm{solar}} = 7.0\times10^{-5}\,\mathrm{eV}^2$. The mixing angles are in the ranges of $0.3 \le \sin^2\theta_{\mathrm{atm}} \le 0.7$ and $0.29 \le \tan^2\theta_{\mathrm{solar}} \le 0.63$. Also the CHOOZ experiment [4] gives an upper bound of 0.22 on the ν_e–ν_x (where ν_x can be either ν_μ or ν_τ or a linear combination) oscillation parameter for $\Delta m^2_{x1} = |m_x|^2 - |m_{\nu_1}|^2 > 10^{-3}\,\mathrm{eV}^2$.

In the model discussed here the atmospheric neutrino and K2K data can be explained by oscillation between the muon and the tauon neutrinos, and the solar neutrino and KamLAND data are explained by oscillation between the electron and muon (or a linear combination of muon and tauon neutrino) neutrinos. In this case the CHOOZ limit applies to the oscillation between the electron and tauon neutrinos[1].

Setting f_2^{ϕ} in (3) to 0, one obtains the famous Zee-Wolfenstein mass matrix,

$$M_\nu = \begin{pmatrix} 0 & \tilde{a} & \tilde{b} \\ \tilde{a} & 0 & \tilde{c} \\ \tilde{b} & \tilde{c} & 0 \end{pmatrix}, \tag{4}$$

where $\tilde{a} = Af^{e\mu}(m_\mu^2 - m_e^2)$, $\tilde{b} = Af^{e\tau}(m_\tau^2 - m_e^2)$ and $\tilde{c} = Af^{\mu\tau}(m_\tau^2 - m_\mu^2)$. One can redefine the neutrino and charged lepton phases such that all $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ are real.

Unfortunately, the Zee-Wolfenstein model is now ruled out by the experimental data. This can be seen from the following.

The above mass matrix satisfies the "zero sum" condition $m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0$; therefore all the neutrino masses are determined in terms of the mass-squared differences [16]. We have [16]

$$\begin{aligned} m_{\nu_1}^2 = -\frac{1}{3}\Big[&2\Delta m^2_{21} + \Delta m^2_{32} \\ &- 2\sqrt{\left(\Delta m^2_{32}\right)^2 + \Delta m^2_{21}\Delta m^2_{32} + \left(\Delta m^2_{21}\right)^2}\,\Big] . \end{aligned} \tag{5}$$

The other two masses are given by $m^2_{\nu_2} = \Delta m^2_{21} + m^2_{\nu_1}$ and $m^2_{\nu_3} = \Delta m^2_{32} + m^2_{\nu_2}$. The "zero sum" condition admits two types of mass hierarchy if the absolute value of $r = \Delta m^2_{21}/\Delta m^2_{32}$ is much smaller than 1 (experimentally $|r| < 0.106$ at 3σ level), with one of them the normal one: $m_{\nu_3} > m_{\nu_2} > m_{\nu_1}$ and $m_{\nu_1} \approx m_{\nu_2}$, and another the inverted one: $|m_{\nu_2}| > |m_{\nu_1}| > |m_{\nu_3}|$ and $m_{\nu_2} \approx -m_{\nu_1}$. One finds that $|x| = |m_{\nu_1}/m_{\nu_2}|$ is determined to be very close to 1.

The mass matrix element $M_{11} = 0$ leads to $V_{e1}^2 m_{\nu_1} + V_{e2}^2 m_{\nu_2} + V_{e3}^2 m_{\nu_3} = 0$, which can be rewritten as

$$V_{e2}^2 = \frac{-x + (1+2x)V_{e3}^2}{1-x}\,. \tag{6}$$

Since $|x|$ is smaller than, but close to 1, the above equation only allows for negative x for small V_{e3}^2, implying that only the inverted mass hierarchy is possible. One thus obtains a minimal $V^2_{e2,\mathrm{min}}$ of V^2_{e2} close to $(1 - V^2_{e3,\mathrm{max}})/2 \approx 0.47$, while the data from SNO and KamLAND prefer a smaller V_{e2}^2. Therefore SNO and KamLAND data disfavor the Zee-Wolfenstein model. This has been noticed in [11]. However, we would like to point out that although the Zee-Wolfenstein model cannot produce the central values for the mixing and mass difference from solar and KamLAND data, the present data cannot rule out the model at more than even the 2σ level.

To have a more quantitative statement, we have carried out a detailed study and the results are shown in Fig. 1. The dashed lines in Fig. 1 are for $\tan^2\theta_{\mathrm{solar}}$ ($\sin^2 2\theta_{\mathrm{solar}} = 4|V_{e1}|^2|V_{e2}|^2$) with two values (0.22 and 0.15) of V_{e3} as a function of r. When $|V_{e3}|$ decreases, $\tan^2\theta_{\mathrm{solar}}$ increases. $\tan^2\theta_{\mathrm{solar}}$ is about 0.53 for the 3σ upper bound of $|V_{e3}|$, and becomes larger than the 3σ allowed value of 0.63 when $|V_{e3}|$ decreases to lower than 0.15. One therefore can take $|V_{e3}|$ to be larger than 0.15 at the 3σ level. It is clear that the model does not have the possibility to produce the best fit value of 0.4 for $\tan^2\theta_{\mathrm{solar}}$. However, at 2σ, $\tan^2\theta_{\mathrm{solar}}$ can be as large as 0.54 [14]. Therefore it is not possible to rule out the model at more than 2σ level from the data on solar neutrinos and KamLAND.

Data on $\sin^2\theta_{\mathrm{atm}}$ can provide further constraints on the model. The condition $M_{22} = m_1 V_{\mu1}^2 + m_2 V_{\mu2}^2 + m_3 V_{\mu3}^2 = 0$ in the model can be used to determine $\sin^2\theta_{\mathrm{atm}} = V_{\mu3}^2$.

[1] There is additional evidence for oscillation between electron and muon neutrinos from the LSND experiment [15]. It confirmed that more neutrinos are needed to explain all the data.

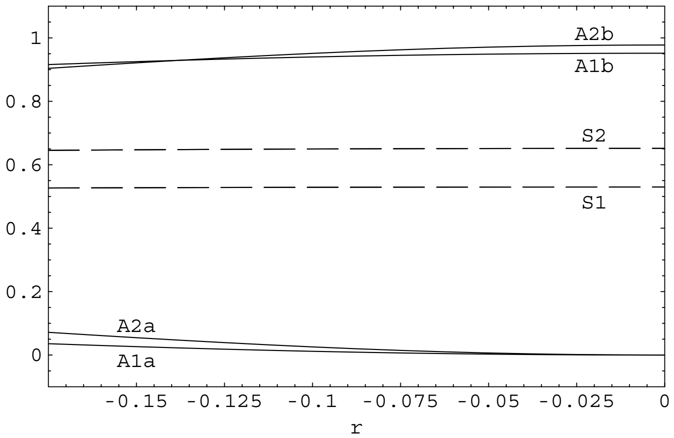

Fig. 1. The dashed lines $S1$ and $S2$ are for $\tan^2\theta_{\rm solar}$ as functions of r. The two solutions for $\sin^2\theta_{\rm atm}$ are indicated by solid lines $A1a$ and $A2a$, and $A1b$ and $A2b$, respectively. Here the indices "1" and "2" indicate the cases with $|V_{e3}|$ equal to 0.22 and 0.15, respectively

The mixing matrix V can be parameterized using three rotation angles, for example [1] $V_{e2} = s_{12}c_{13}$, $V_{e3} = s_{13}$ and $V_{\mu 3} = s_{23}c_{13}$. Here $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Two of the angles, $\theta_{12,13}$, can be determined in terms of V_{e3} and r from previous discussions. The condition

$$\begin{aligned} M_{22} &= m_{\nu_1}(s_{12}+c_{12}s_{13}t_{23})^2 + m_{\nu_2}(c_{12}-s_{12}s_{13}t_{23})^2 \\ &\quad + m_{\nu_3}c_{13}^2 t_{23}^2 \\ &= 0 \end{aligned} \tag{7}$$

then determines θ_{23} in terms of V_{e3} and r. Here $t_{23} = s_{23}/c_{23} = \tan\theta_{23}$. There are two solutions for $\tan\theta_{23}$ for given V_{e3} and y which we indicate by "a" and "b".

In Fig. 1 the solid lines show $\sin^2\theta_{\rm atm}$ as a function of r for $|V_{e3}|$ equal to its 3σ allowed upper value of 0.22 and the allowed lower value of 0.15. From the figure we see that $\sin^2\theta_{\rm atm}$ decreases for solution "a", and $\sin^2\theta_{\rm atm}$ increases for solution "b" as r increases from the 3σ lower bound of -0.106 to the allowed upper bound of 0. All solutions for $\sin^2\theta_{\rm atm}$ are outside the 3σ allowed range of $0.3 \sim 0.7$. For $|V_{e3}|$ smaller than 0.15, it is possible for $\sin^2\theta_{\rm atm}$ of solution "b" to become smaller than the 3σ allowed upper bound. However, $|V_{e3}|$ smaller than 0.15 will drive $\tan^2\theta_{\rm solar}$ out of the 3σ allowed range. Therefore the combined neutrino data on $\tan^2\theta_{\rm solar}$ and $\sin^2\theta_{\rm atm}$ rule out the Zee-Wolfenstein model at more than 3σ level.

The above discussion clearly shows that the Zee-Wolfenstein neutrino mass matrix is in trouble. That does not, however, mean that the Zee model itself is in trouble. The mass matrix given in (3) contains more than enough freedom to fit the data. Here we encounter a common problem for physics beyond the SM: that there are too many new parameters. Additional theoretical considerations have to be applied to narrow down the parameters.

We find that an interesting neutrino mass matrix emerges if one requires that there should be no large hierarchies among the new couplings; that is, all f^{ij} and $f_2^{\phi,ab}$ are of the same order of magnitude, respectively. This can be considered as a naturalness requirement. From (3) one sees that all terms in the mass matrix are either proportional to m_l or m_l^2. Since $m_\tau >> m_{\mu,e}$, the leading contributions to the neutrino mass matrix are proportional to $f^{ij}m_\tau^2$ and $f_2^{\phi,ab}m_\tau$. To this order we have

$$\begin{aligned} M_{11} &= -2A\frac{v}{\cos\beta}f^{e\tau}f_2^{\phi,\tau e}m_\tau\,, \\ M_{22} &= -2\frac{v}{\cos\beta}f^{\mu\tau}f_2^{\phi,\tau\mu}m_\tau\,, \\ M_{33} &= 0\,, \\ M_{12} &= -\frac{v}{\cos\beta}A(f^{e\tau}f_2^{\phi,\tau\mu}+f^{\mu\tau}f_2^{\phi,\tau e})m_\tau\,, \\ M_{13} &= Af^{e\tau}m_\tau\left(m_\tau - \frac{v}{\cos\beta}f_2^{\phi,\tau\tau}\right)\,, \\ M_{23} &= Af^{\mu\tau}m_\tau\left(m_\tau - \frac{v}{\cos\beta}f_2^{\phi,\tau\tau}\right)\,. \end{aligned} \tag{8}$$

Without loss of generality, by appropriate choices of the neutrino field phases, the 11, 13, 23 entries can be made real with just one physical phase δ in the mass matrix. One can rewrite the above mass matrix as

$$M_\nu = a\begin{pmatrix} 1 & (y\mathrm{e}^{\mathrm{i}\delta}+x)/2 & z \\ (y\mathrm{e}^{\mathrm{i}\delta}+x)/2 & xy\mathrm{e}^{\mathrm{i}\delta} & xz \\ z & xz & 0 \end{pmatrix}\,, \tag{9}$$

with $a = |M_{11}|$, $x = |f^{\mu\tau}|/|f^{e\tau}|$, $y = |M_{22}|/xa$, $z = |M_{13}|/a$.

This is a highly constrained form of the mass matrix. This matrix is rank 2, implying that one of the neutrinos has zero mass. The non-zero eigenvalues are given by

$$\begin{aligned} m_\pm^2 = \frac{a^2}{4}\Big(&\sqrt{1+2xy\cos\delta+x^2+y^2} \\ &\pm\sqrt{(1+x^2)(1+y^2+4z^2)}\Big)^2\,. \end{aligned} \tag{10}$$

Since experimentally $\Delta m_{21}^2 > 0$, there are two types of eigen-mass hierarchies,

(a) $m_{\nu_1} = 0\,,\quad |m_{\nu_2}| = \sqrt{\Delta m_{21}^2} = m_-\,,$

$|m_{\nu_3}| = \sqrt{\Delta m_{32}^2 - \Delta m_{21}^2} = m_+\,;$ and

(b) $|m_{\nu_1}| = \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2} = m_-\,,$

$|m_{\nu_2}| = \sqrt{|\Delta m_{32}^2|} = m_+,\quad m_{\nu_3} = 0\,.$

The five parameters in the mass matrix are severely constrained by the data on $\Delta m_{21,32}^2$, V_{e2}, V_{e3} and $V_{\mu 3}$.

To have some idea about what parameter space may satisfy the experimental constraints, let us discuss the situation with the phase δ set to 0 for simplicity. For the type (a) of mass hierarchy, since $|r| = |\Delta m_{21}^2/\Delta m_{32}^2|$ is much smaller than 1, one would have $(1+xy)^2$ to be almost equal to $(1+x^2)(1+y^2+4z^2)$. To satisfy this, x should be close to y and z to be much smaller than 1. Expanding the mixing matrix elements around $x = y$ and small z, we find $(V_{e2},\ V_{\mu 2},\ V_{\tau 2})$ to be proportional to $(z,\ xz,\ -(1+x^2))$. Since z is much smaller than 1, one would obtain too small a V_{e2}, in contradiction with the solar and KamLAND data. This qualitative feature is not changed even if a non-zero δ is introduced. There is no solution for the normal mass hierarchy of type (a).

For the type (b) of mass hierarchy, $(V_{e3}\ V_{\mu 3},\ V_{\tau 3})$ is proportional to $(-2xz,\ 2z, x-y)$. A small $|r|$ requires xy to be close to -1. The small ratio $|V_{e3}|/|V_{\mu 3}| = x$ requires $|x|$ to be less than about 0.3 which can be taken as a starting value for x for numerical analysis. We find that $xy+1>0$ does not have phenomenologically acceptable values to fit the data, there being a too large $\tan^2\theta_{\rm solar}$. We have surveyed a wide range for the parameters x, y and z, and find that it is not possible to get a set of values such that they can produce the central values for r, $\tan^2\theta_{\rm solar}$ and $\sin^2\theta_{\rm atm}$ without violating the 3σ upper bound of V_{e3}. This can be understood as follows. To get a small V_{e3} one requires a small x. The requirement of xy to be close to -1 results in a large y which dictates a large $V_{\tau 3}\sim x-y$ and leads to a too small $V_{\mu 3}$ to explain the atmospheric neutrino and K2K data. There are correlations between V_{e3} and $\tan^2\theta_{\rm solar}$, V_{e3} and r. When $V_{e3}(x)$ decreases, $\tan^2\theta_{\rm solar}$ increases, and r decreases. These correlations also constrain the parameters. We however find regions of parameters such that r, $\tan^2\theta_{\rm solar}$ and $\sin^2\theta_{\rm atm}$ are within their 1σ regions, while V_{e3} is less than the 90% C.L. allowed region. In the following we present a sample solution given by

$$\begin{aligned}
&m_{\nu_1} = 4.92\times 10^{-2}\,\mathrm{eV},\quad m_{\nu_2} = -5.00\times 10^{-2}\,\mathrm{eV}\,,\\
&m_{\nu_3} = 0\,,\\
&V = \begin{pmatrix} 0.8121 & -0.5615 & -0.11584\\ 0.3495 & 0.6856 & -0.6387\\ 0.4672 & 0.4633 & 0.7530\end{pmatrix}\,. \qquad (11)
\end{aligned}$$

The corresponding values for r, $\tan^2\theta_{\rm solar}$ and $\sin^2\theta_{\rm atm}$ are 0.0317, 0.0478 and 0.408, respectively. They are all within their 1σ allowed regions. The value -0.1584 for V_{e3} is below the 90% C.L. allowed range.

In the above solution, the input parameters are $x = -0.48$, $y = 4,323$, $z = 1.9$, $a = 1.67\times 10^{-2}$ eV. One can choose different signs for the parameters x, y and z. As long as the signs for x and y are simultaneously changed, the magnitudes of the eigen-masses and the mixing matrix elements are not changed. We will stick to the signs with x negative, y and z positive in our later discussions.

One can also find solutions with smaller $|V_{e3}|$. For example, with $x = -0.165$, $y = 6.531$ and $z = 3.030$, we obtain $V_{e3} = -0.11$, but $\tan^2\theta_{\rm solar} = 0.624$ is too close to the 3σ bound.

We searched for other solutions. We find that it is also possible to have solutions with non-zero CP violating phase δ. For example with $a = 1.92\times 10^{-2}$ eV, $x = -0.276$, $y\mathrm{e}^{\mathrm{i}\delta} = 3.467 - \mathrm{i}0.0573$, and $z = 1.571$, we have

$$\begin{aligned}
&m_{\nu_1} = 4.93\times 10^{-2}\mathrm{e}^{-\mathrm{i}16.9^\circ}\,\mathrm{eV}\,,\\
&m_{\nu_2} = -5.00\times 10^{-2}\mathrm{e}^{\mathrm{i}11^\circ}\,\mathrm{eV}\,,\quad m_{\nu_3} = 0\,, \qquad (12)\\
&V = \begin{pmatrix} 0.8147 & -0.5048-\mathrm{i}0.2311 & -0.1676-\mathrm{i}0.0024\\ 0.3110-\mathrm{i}0.1995 & 0.7035 & -0.6071-\mathrm{i}0.0087\\ 0.4166-\mathrm{i}0.1619 & 0.4402-\mathrm{i}0.0561 & 0.7767\end{pmatrix}\,.
\end{aligned}$$

For the above input parameter set, $\tan^2\theta_{\rm solar}$, $\sin^2\theta_{\rm atm}$, r and $|V_{e3}|$ are all in their 2σ allowed ranges. The CP violating Jarlskog parameter $J = \mathrm{Im}(V_{11}V_{22}V_{12}^*V_{21}^*)$ is predicted to be -0.0165 which may be studied in future neutrino factories. We have kept the masses in the form with phases to illustrate the existence of Majorana phases which can be rotated away by multiplying a phase matrix from the right on V as obtained above.

The neutrino masses obtained in the model are in the interesting ranges. The sum of the absolute neutrino masses, $m_{\rm sum} = |m_{\nu_1}|+|m_{\nu_2}|+|m_{\nu_3}|$, in this model is around 0.1 eV which is several times smaller than the recent bound of 0.69 eV from WMAP [17], but can be probed in the near future by the PLANK experiment where the sensitivity on $m_{\rm sum}$ can be as low as 0.03 eV. Laboratory neutrino mass experiments can also test the model. A non-zero value $a = |m_{ee}|$ can induce neutrinoless double beta decays. $|m_{ee}|$ obtained here is about 0.02 eV which is safely below the present bound [1,18] of 0.4 eV. However it can be probed by future experiments, such as GENIUS, MOON and CUORE, where a sensitivity of about 0.01 eV may be reached. The effective mass

$$\langle m_e\rangle = \sqrt{|m_{\nu_1}V_{e1}|^2 + |m_{\nu_2}V_{e2}|^2 + |m_{\nu_3}V_{e3}|^2}$$

measured by the end point spectrum of beta decay in our case is around ~ 0.05 eV, which is unfortunately a factor of 2 smaller than the sensitivity of the future KATRIN experiment.

The off-diagonal entries of the couplings f^{ab} and $f_2^{\phi,ab}$ can induce flavor changing interactions. One should make sure that the constraints on the related parameters will not rule out the regions of the parameters to produce the mass matrix discussed above. It is not possible to completely determine the couplings using just information from neutrino masses and mixing. We therefore take a simple situation with $f_2^{\phi,\tau\tau} = 0$ for illustration. In this case for the example given in (12): $f_2^{\phi,\tau e}/\cos\beta = -0.33\times 10^{-2}$, $f_2^{\phi,\tau\mu}/\cos\beta = -(1.14-\mathrm{i}0.02)\times 10^{-2}$, $Af^{e\tau} = 0.93\times 10^{-11}(\mathrm{GeV}^{-1})$, and $Af^{\mu\tau} = -0.26\times 10^{-11}(\mathrm{GeV}^{-1})$. It is interesting to note that the solution obtained here is consistent with the naturalness requirement that $f_2^{\phi,\tau e}$ is of the same order of magnitude as $f_2^{\phi,\tau\mu}$, and $f^{e\tau}$ is of the same order of magnitude as $f^{\mu\tau}$. If one chooses a smaller x one would obtain a bigger hierarchy for the parameters, $f^{\mu\tau}$ and $f^{e\tau}$. The qualitative features will not change when other values for the parameters are used.

Exchange of the neutral Higgs boson ϕ_1 (with mass M_0) can induce at tree level $l_i \to l_j l_k \bar{l}_k$ decays. At one loop level $l_i \to l_j\gamma$ can also be induced. The branching ratios for the classes of the decays are given by

$$B(l_i \to l_j l_k \bar{l}_k)$$
$$= (|f_2^{\phi,ij}|^2 + |f_2^{\phi,ji}|^2)\frac{(m_k/v)^2}{(G_F M_0^2 \tan\beta \sin\beta)^2}$$
$$\times B^{SM}(l_i \to \nu_i l_k \bar{\nu}_k)\,,$$
$$B(l_i \to l_j \gamma) \tag{13}$$
$$= \alpha_{em}(|f_2^{\phi,ij}|^2 + |f_2^{\phi,ji}|^2)\frac{(m_i/v)^2}{(G_F M_0^2)^2} B^{SM}(l_i \to \nu_i l_k \bar{\nu}_k)\,.$$

where B^{SM} indicates the branching ratio predicted by the SM.

For the values of $f_2^{\phi,\tau\mu}$ and $f_2^{\phi,\tau e}$ obtained in the example of (12) we have

$$B(\tau \to \mu\mu\bar{\mu}, \mu e\bar{e}) \approx 3.5 \times 10^{-9} B_\tau,\ 0.80 \times 10^{-13} B_\tau\,;$$
$$B(\tau \to \mu\gamma) \approx 0.76 \times 10^{-8} B_\tau\,;$$
$$B(\tau \to e\mu\bar{\mu}, ee\bar{e}) \approx 2.9 \times 10^{-10} B_\tau,\ 0.67 \times 10^{-14} B_\tau\,;$$
$$B(\tau \to e\gamma) \approx 0.63 \times 10^{-9} B_\tau\,.$$

In the above $B_\tau = (100(\mathrm{GeV})/M_0 \tan\beta)^4 B^{SM}(\tau \to \nu_\tau \mu \bar{\nu}_\mu)$ with $B^{SM}(\tau \to \nu_\tau \mu \bar{\nu}_\mu) \approx 17\%$.

There are experimental constraints on the above decays with the 90% C.L. bounds, given by [1], $B(\tau \to \mu\mu\bar{\mu}, \mu e\bar{e}) = 1.9 \times 10^{-6}$, 1.7×10^{-6}, $B(\tau \to e\mu\bar{\mu}, ee\bar{e}) = 1.8 \times 10^{-6}$, 2.9×10^{-6}, $B(\tau \to \mu\gamma, e\gamma) = 1.1 \times 10^{-6}$, 2.7×10^{-6}. For $\tan\beta$ of order 1, all the branching ratios predicted above are safely below the experimental values if the mass M_0 is of order 100 GeV.

A non-zero f^{ij} can also induce radiative charged lepton decays by exchanging charged scalars with the branching ratio given by

$$B(l_i \to l_j \gamma) = \frac{\alpha_{em}}{48\pi (G_F \bar{M}_h^2)^2} (f^{ik} f^{jk})^2\,. \tag{14}$$

Here$\bar{M}_h^2 = \cos^2\theta_Z/M_1^2 + \sin^2\theta_Z/M_2^2$. If the parameter A is not too much smaller than the natural value of $A = 10^{-5}$ (GeV^{-1}), their contributions for the rare decays mentioned will be much smaller. The rare decays of these types will not provide significant constraints.

From the above discussions we see that the new form of the mass matrix proposed is consistent with the present experimental data (within 90% allowed regions). It also predicts $m_{\nu_3} = 0$ and a sizeable $|V_{e3}|$. If the error on $\tan^2\theta_{solar}$ is reduced and the present best fit value holds, $|V_{e3}|$ will be close to the 3σ allowed upper bound. This model can be tested in the future.

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